### Intergenerational Equitable Climate Change Mitigation: Negative Effects of Stochastic Interest Rates; Positive Effects of Financing

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### Abstract

Today's decisions on climate change mitigation affect the damage that future generations will bear. Discounting future benefits and costs of climate change mitigation is one of the most critical components of assessing efficient climate mitigation pathways. We extend the DICE model with stochastic discount rates to reflect the uncertain nature of discount rates. Stochastic rates give rise to a stochastic mitigation strategy, resulting in all model quantities becoming stochastic.

We show that the classical calibration of the DICE model induces intergenerational inequality: future generations have to bear higher costs relative to GDP. Further, we show that considering stochastic discount rates and stochastic abatement policies, which can be interpreted as successive re-calibration, increases intergenerational inequality (and adds additional risks). Motivated by this, we consider additional financing risks by investigating two modifications of DICE. We find that allowing financing of abatement costs and considering non-linear financing effects for large damages improves intergenerational effort sharing. To conclude our discussion of options to improve intergenerational equity in an IAM, we propose a modified optimization to keep costs below 3 % of GDP, resulting in more equal distribution of efforts between generations.

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### 1 Introduction

Climate change is one of the most significant risks of the next centuries [1], as demonstrated by its manifold impacts on society [2]. Due to the persistent nature - on human time scales - of the main greenhouse gas carbon dioxide [3], a transition to (net) zero emissions is necessary to limit global warming to any constant level. A diverse set of approaches have been developed to find the optimal pathway towards such an emission-neutral world. One traditional method for the assessment of transition pathways are integrated assessment models (IAMs) of the climate and economic systems; for an overview of the developments on IAMs see [4, 5].

We focus on the DICE model [6, 7] as a simple IAM, which is frequently used to demonstrate the impacts of modifications to the original model due to its simplicity [8, 9, 10, 11, 12]. While IAMs were first set up as deterministic, stochastic shocks have been included to consider the risk of tipping points [13, 14, 15, 16, 17, 18, 19], natural feedback processes such as permafrost [9] or abstract catastrophic risks [20, 21]. Since most damages of climate change will occur over long time scales, value-based decisions on the discount rate and optimization function have a strong influence on the resulting optimal pathways [22, 23, 24, 25, 26]. Stochastic approaches also require more advanced risk evaluations than Monte Carlo averaging to capture the full extent of tail risks [13] or inequalities in the distribution of damages [27].

Most of these contributions aim to provide an estimate of the societal costs of emitting one additional ton of carbon dioxide, the so-called social cost of carbon (SCC), which is an important determinant for benefit-cost analysis of climate mitigation. Model extensions updating IAMs [11, 12, 19, 28] show a wide range of potential SCC, while also econometrics-based SCC estimates for single impacts such as mortality [29] or energy consumption [30], but also general SCC [31] emphasize the need to improve the precision of the estimated SCC, especially by including, so far ignored, additional impact channels. Discount rates, in particular, are an essential component of IAM modelling. Thus, the discussion between proponents of a descriptive approach based on observation of market returns like in DICE [7] versus a normative approach to discounting as in the Stern review [22] continues. The assumptions on discounting have already been discussed in the context of intergenerational [32] and intragenerational inequality [33]. Recent studies [34, 35] show that dynamic modelling of discount rates might allow a more precise assessment of the costs of damages and abatement. While our focus lies on the effect of interest rates on intergenerational equity, we acknowledge that intergenerational equity can be tackled with other policy extensions [36], e.g. carbon taxation, [37].

Parallel to this work, an updated DICE-2023 model was developed and published [38] - one of the most prominent updates includes the consideration of uncertainty in the discount rate through the inclusion of consumption growth uncertainty and riskiness of climate investments. Since the final model remains with a deterministic expected discount rate, our contribution remains independent considering extensions with funding mechanisms and a stochastic discount rate model.

We examine the calibration of the DICE model with respect to intergenerational equity. Measuring the costs per GDP of the optimal (calibrated) climate emissions path in the standard model, we find that these costs are unequally distributed across generations. We use an implementation of the DICE model with a flexible time discretization [39] and couple this model with a comprehensive modelling toolbox for financial markets [40] to enable the integration of valuation methods for complex financial products in IAMs. We add stochastic interest rates to the model and introduce a stochastic abatement policy. The stochastic abatement policy mimics the process of reacting to changes in the interest rate level by repeated (over time) re-calibration of the model. Since intergenerational inequality is related to the sensitivity of the utility and the discount factor, we introduce two model extensions related to these two parts that influence intergenerational equity: Financing of abatement costs and non-linear discounting of costs, [41]. As abatement is a planned investment process, we assume that these costs may be financed by loans with stochastic interest rates. Furthermore, as large projects and investments often incur additional costs [42, 43], we introduce increasing financing costs for large payments via a non-linear discounting. These higher costs for high damages can be viewed as (additional) financing risks. We provide an overview of the extensions in Fig. 1. More background is provided in the methods section.

Limiting total cost by GDP share					
Stochastic interest rates	Funding of abatement cost	Non-linear funding of large damage cost			
Basic model DICE-2016-R [7] – re-implementation with improvements					
• Integrating numerical methods[40] (Monte-Carlo simulation, algorithmic differentiation)					
<ul> <li>Improved representation of time-discretisation via an Euler-scheme.</li> </ul>					

**Figure 1: Interest rate related model extensions.** We expand our re-implementation of DICE-2016-R with 1) a stochastic interest rate module with an optional stochastic abatement model, 2) the option to allow funding of abatement cost, and, 3) the option to apply non-linear discounting of damage cost. The latter can be used to penalize emission paths that generate cost overruns in terms of cost-per-GDP.

### 2 Results

#### 2.1 Intergenerational Inequality

To find the optimal mitigation pathway, the standard DICE model equates the *increment* of the cost of damage  $dC_D(t)$  to the *increment* of the cost of abatement  $dC_A(t)$ , both weighted by the welfare-cost-sensitivity dV/dC and the discount factor  $\frac{N(0)}{N(t)}$ ,

$$\int_0^T \frac{\mathrm{d}V(t)}{\mathrm{d}C(t)} \frac{\mathrm{d}C_{\mathrm{D}}(t)}{\mathrm{d}\mu(s)} \frac{N(0)}{N(t)} \mathrm{d}t + \int_0^T \frac{\mathrm{d}V(t)}{\mathrm{d}C(t)} \frac{\mathrm{d}C_{\mathrm{A}}(t)}{\mathrm{d}\mu(s)} \frac{N(0)}{N(t)} \mathrm{d}t \stackrel{!}{=} 0.$$
(7)

Considering a simplified 1-parameter abatement model parameterizing the abatement policy by  $T^{\mu=1}$ , the time by which 100% abatement is achieved, this simplifies to

$$\int_{0}^{T} \frac{dV(t)}{dC(t)} \frac{dC_{D}(t)}{dT^{\mu=1}} \frac{N(0)}{N(t)} dt + \int_{0}^{T} \frac{dV(t)}{dC(t)} \frac{dC_{A}(t)}{dT^{\mu=1}} \frac{N(0)}{N(t)} dt \stackrel{!}{=} 0.$$
(9)

The conditions (7) and (9) are plausible if a single agent optimizes its cost but may be problematic if the costs are distributed between different generations. Note that the two individual contributions in eq. (9) occur at different magnitudes at different times, even different generations. Fig. 2 shows the temporal distribution of the weighted functions  $\frac{dC_A(t)}{dT^{\mu=1}}$  and  $\frac{dC_D(t)}{dT^{\mu=1}}$ .

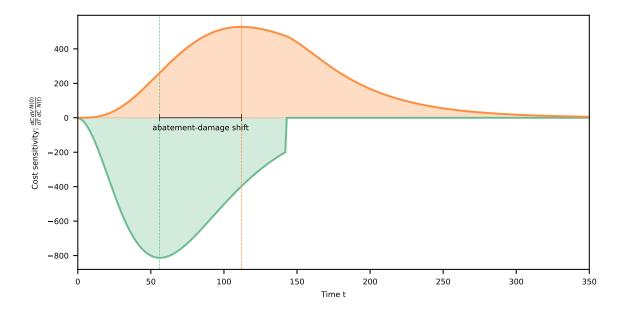
Hence, *marginal* gains of one generation are equated to *marginal* losses of another generation. The calibration does not equalize the burden but the weighted marginal burden. In this sense, the calibration is indifferent to the temporal distribution of the absolute burden. This may generate intergenerational inequality.

#### 2.2 Including stochastic interest rates into DICE

We see in Equation (7) that both the welfare-cost-sensitivity  $\frac{dV(t)}{dC(t)}$  and the numeraire ratio  $\frac{N(0)}{N(t)}$  (i.e., the discount factor) are important to determine the inter-temporal optimal pathway.

Including a stochastic interest rate model, the numéraire N becomes stochastic. Allowing for (stochastic) adaptation of the abatement policy  $\mu$  to the interest rate level, all model quantities become stochastic.

We use a classical Hull-White model - details are provided in the methods (11). The model generates the same expected discount rate as for the deterministic DICE

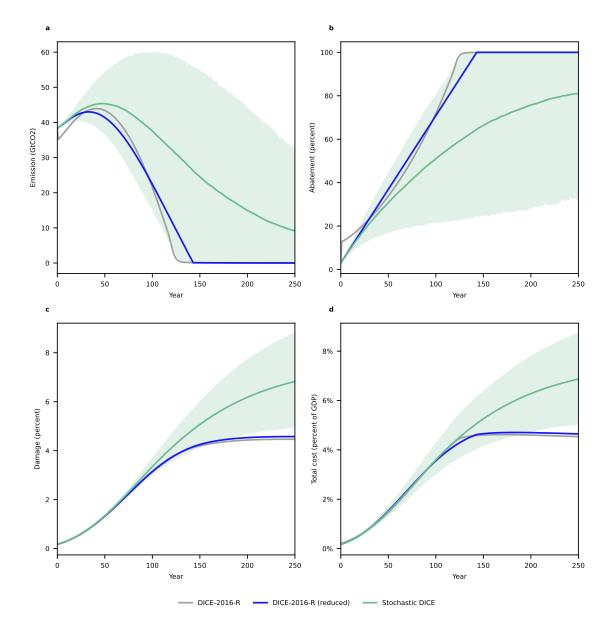


**Figure 2:** The sensitivity  $\frac{dC_A(t)}{dT^{\mu=1}}$  and  $\frac{dC_D(t)}{dT^{\mu=1}}$  of abatement cost  $C_A$  and damage cost  $C_D$  on changes of the abatement policy  $T^{\mu=1}$ . The calibration is balancing the two areas under/above these curves - which is ultimately *not* resulting in an intergenerational balancing of cost, as visible by the shift between the maxima.

model we compare it with, which implies that combining the stochastic interest rates with a deterministic abatement model results in the same emission pathways as for the deterministic model. As is well known, from the literature, the level of the discount rate has a large influence on optimal abatement paths - we show in the supplementary information, that an increasing interest rate level defers abatement, while increasing volatility speeds up abatement, as we further discuss in [39].

However, allowing for a stochastic abatement policy that adapts to the interest rate level, e.g., under the model (12) presented in the methods, we see an adverse effect: the expected abatement is smaller, the expected emissions are higher and the expected total cost increase significantly for later years. Note that restricting the stochastic abatement model (12) to the case  $a_1 = 0$  agrees with the abatement model (4). The effect is hence solely created by allowing the change of abatement speed by interest rate change  $a_1$  to be a free parameter.

Fig. 3 depicts the results of stochastic interest rates, both in expectation and in a percentile measure, on emissions, optimal abatement, damage and total cost-per-GDP. These quantities are depicted for the classical deterministic DICE model (grey)



and for the DICE model with stochastic interest rates and stochastic abatement policy (green), where we also show the 10th and 90th percentile values.

**Figure 3:** Classical DICE (grey), Classical DICE with reduce Abatement Model (4) (blue) and Stochastic DICE Model (green). Restricting the abatement policy to the 1-parameter model (4) has a comparably small effect on the emission pathways. Allowing adaptation of the abatement policy to stochastic interest rates increases intergenerational inequality *and* exhibits a significant risk of even higher intergenerational inequality (**a**: emissions, **b**: abatement, **c**: damages, and **d**: total GDP relative cost). Shaded area shows the 10th to 90th percentiles.

This shows that under the stochastic interest rate and stochastic abatement model, the optimal policy will increase intergenerational inequality and, in addition, exhibit the risk of further increases in intergenerational inequality (as given by the percentiles). It also lowers the abatement and increases the emission.

Such an effect is known under the name of "convexity" in the context of the valuation of financial derivatives: While (10) shows that stochastic interest rates will result in the same cost and emission paths for the deterministic abatement strategy, we do not recover these cost and emissions for the stochastic abatement strategy as cost and emission are convex functions in the interest rates. Note that for a convex function *f* and a random variable *X* we have  $E(f(X)) \leq f(E(X))$  (Jensen's inequality).

### 2.3 Including funding of abatement and funding risks of damage into DICE

As shown in Fig. 3, costs are distributed unevenly across generations, with higher total costs relative to GDP for generations in the far future.

We investigate two model extensions that alter the distribution of cost and thus will influence intergenerational equity: First, we modify the abatement costs by allowing these to be funded over time for interest that has to be paid. Second, we include a non-linear dependency of discounting of the damage cost on the magnitude of the cost. This can be viewed as the additional cost associated with financing risk of very large amounts [41]. Details on this approach and parameters are provided in the methods section 4.3.4.

We find that both modifications improve intergenerational equity in the classical deterministic DICE model, Fig. 4, and in the DICE model with stochastic interest rate, Fig. 5. In the latter, intergenerational equity is improved with respect to the expectation and the uncertainty in the respective optimal emission pathway, represented by the 10th and 90th percentiles.

Due to the limits of scaling up abatement immediately and the inherent preference for current generations in the DICE model, present generations still face the lowest costs.

Figures 4 and 5 depict the instantaneous cost-per-GDP (C(t)/GDP(t)). For an assessment of the generational burdens we consider the lifetime-average value using

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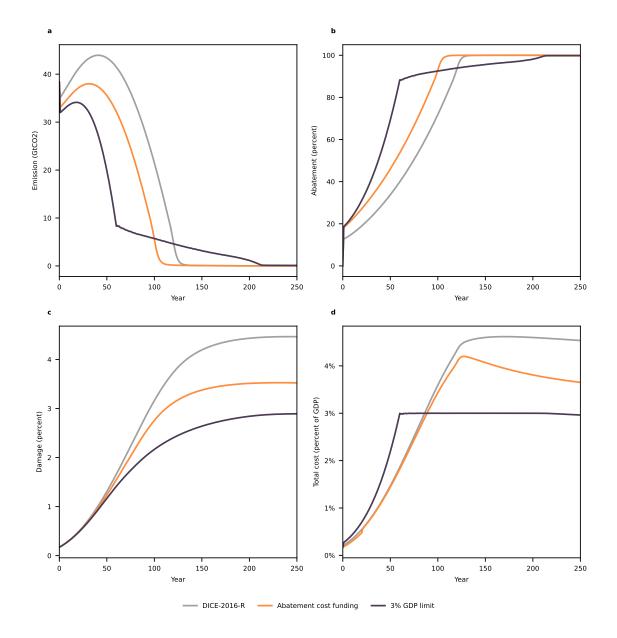


Figure 4: Deterministic model: Financing extensions lead to faster abatement compared to standard model a emissions, b abatement, c damages, and d total GDP relative cost.

population projections from 2015 to 2100 [48] to calculate burdens experienced during the projected lifetime.

We measure the burden in percentage-of-GDP (section 4.2.3), where we choose 3 % as the neutral level (green). While differences are small for the time covered by population projections (Fig. 6 and Fig. 7), extrapolation using the fixed 2100 life expectancy shows a clear divergence at the end of the century. Notably, the stochastic

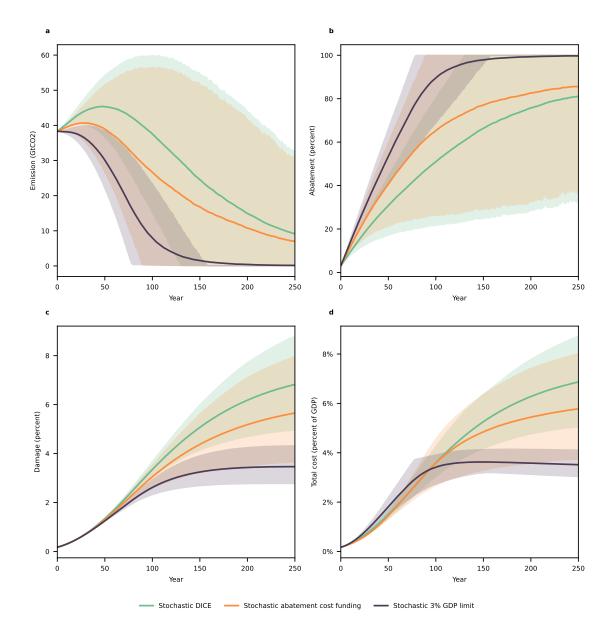
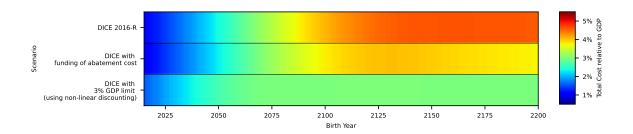
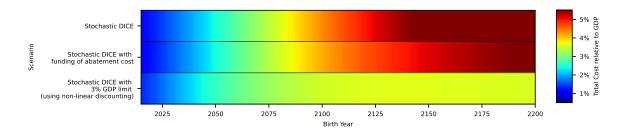


Figure 5: Considering additional costs leads to faster abatement compared to standard model a emissions, b abatement, c damages, and d total GDP relative cost. Shaded area shows the 10th to 90th percentiles.

variant does not stay exactly below 3% of GDP on average since high-risk scenarios are not fully offset. The deterministic model keeps the 3% limit and illustrates the equal burden between generations enabled by imposing GDP relative cost limits.



**Figure 6: Illustration of generational inequality, deterministic model:** Lifetimeaverage cost relative to GDP by year of birth in the different model specifications up to 2200. Normalized by annual GDP and scaled such that blue means lower, red higher average total cost values than 3% of GDP for this generation.



**Figure 7: Illustration of generational inequality, stochastic model:** Lifetimeaverage cost relative to GDP by year of birth in the different model specifications up to 2200. Normalized by annual GDP and scaled such that blue means lower, red higher average total cost values than 3% of GDP for this generation.

### 3 Discussion

We analyze the DICE integrated assessment model and derive metrics to assess intergenerational equity of the optimal emission pathways calibrated by the model. We extend the DICE integrated assessment model and compare several extensions for their effect on intergenerational equity. Our analyses apply to integrated assessment models in general and could also be extended to benefit-cost analysis of climate mitigation. We provide a complete open-source implementation of the model illustrating all our extensions, utilizing Monte-Carlo implementation of standard interest rate models for simulation and (stochastic) algorithmic differentiation to efficiently calculate sensitivities of model quantities against each other.

Our first observation is that the DICE model can significantly increase the policy cost measured in terms of cost-per-GDP. This is because the calibration does not consider any (cost-related) intergenerational equity but balances the marginal aggregated cost changes induced by a policy change. Absolute cost and its distribution over time are irrelevant to the objective function. This effect is driven by the aggregation of discounted welfare into intergenerational welfare.

Considering stochastic interest rates, this effect is amplified when the stochastic adaptation of the abatement strategy generates a significant risk of increased cost. This indicates that a repeated re-calibration of the DICE model to current interest rate levels will amplify intergenerational inequality.

We consider natural model extensions by allowing the funding of abatement cost and non-linear discounting of damage cost. Funding of abatement cost may be considered as modelling the backing of abatement costs by government loans. We find that this may have a positive effect on intergenerational equity in terms of the cost distribution as it improves the alignment of abatement and damage cost. The non-linear discounting of damage cost may be considered as modelling financing risk of damage costs, where very large cost have higher financing cost. The nonlinear damage cost can be used as a constrain that allows to effectively limit the generational cost to a specific percentage of the GDP.

Using a simple IAM includes several caveats. First, the representation of damages could be improved [49, 11, 12]. Second, the lack of regional resolution compared to, e.g., econometric damage estimates [50, 51] ignores regional inequalities in exposure and potential distributional effects of projected costs [52]. Since our study aims to

introduce methods such as limiting total costs of climate change by a proportion of GDP to the debate on mitigation pathways, these limitations could be addressed by including the presented concepts in more complex assessments of mitigation pathways [31, 53].

Modifying the DICE model in a modular way, we address the issue that the classical model does not consider equity between generations. The proposed extensions accounting for the funding cost of abatement and increased financing cost of large damages can improve this aspect of the intertemporal optimization. This is amplified when considering stochastic interest rate risk. Constraining the total costs of mitigation and damages relative to GDP could be used as a simple heuristic to ensure intergenerational equity of climate mitigation.

### 4 Methods

The starting point of our investigations and extensions is the 2016 implementation of the classical DICE model [6, 7], since the DICE-2023 description [38] was only published parallel to this research. Our implementation can deal with an arbitrary timediscretization  $\{t_i\}$  and the DICE model is re-defined as being an Euler-discretisation of a time-continuous model. For the equi-distant discretisation with  $\Delta t_i = 5$  years the classical DICE-2016 model is recovered. Our numerical experiments are based on a time-discretization with  $\Delta t_i = 1$  year. The differences to the classical DICE-2016 are negligible for our work.

Except where otherwise stated, we use the same model components (e.g. abatement cost and damage cost model) as in DICE-2016. A detailed description of our implementation of the DICE model (and its extensions) can be found in [39]. We use the same notation, except that we denote the cost functions with C.<sup>1</sup>

We will recall the calibration procedure first in Section 4.1, where we introduce a parameter-reduced form of the abatement policy to use for our analysis.

We then present metrics to analyse the temporal distribution of cost and the temporal distribution of cost-sensitivity to policy changes in the classical DICE model in Section 4.2, since the question at which time cost occur is a question of intergenerational equity.

We conclude presenting different extensions to the model, which change the distribution of cost. We discuss these in Section 4.3. We perform our analysis for the classical (deterministic) DICE model, as well as a model extended with stochastic interest rates and a stochastic abatement policy.

#### 4.1 Model Calibration

The classical objective function of the DICE model is the integrated discounted welfare

$$\int_0^T V(t) \cdot \frac{N(0)}{N(t)} \mathrm{d}t. \tag{1}$$

<sup>&</sup>lt;sup>1</sup>In [7] the letter C was used for consumption, which we do not reference here.

Here *N* denotes the numéraire. For the classical model, this means that N(0)/N(t) is the discount factor. We use the more general notion of a numéraire, as we will consider a more general model later. DICE-2016 is recovered by  $N(t) = \exp(r \cdot t)$ .

#### 4.1.1 Objective Function for the Stochastic Model

In the following we will consider an extension of the model where N and V are stochastic processes. This requires that we re-define the objective function as (1) is a random variable.

For the calibration we consider the expectation operator

$$\mathsf{E}\left(\int_{0}^{T}V(t)\cdot\frac{N(0)}{N(t)}\mathsf{d}t\right),\tag{2}$$

i.e., the objective function for the stochastic model is the expected welfare.

For analysis purposes we will plot a risk measure, here the value-at-risk risk, i.e., a percentile of the stochastic welfare,

$$\operatorname{VaR}_{\alpha}\left(\int_{0}^{T}V(t)\cdot\frac{N(0)}{N(t)}\mathrm{d}t\right).$$
(3)

#### 4.1.2 Abatement Policy - Classical DICE Model

The model calibration determines the optimal abatement policy  $t \mapsto \mu(t)$  and optimal savings rate s(t) that maximizes the integrated discounted welfare (1), (2).

In the time-discrete classical DICE model the calibration determines the optimal parameters  $i \mapsto (\mu(t_i), s(t))$ . For a model time horizon of T = 500 years and a time-discretization step of  $\Delta t_i = 1$  year this results in 499 free parameters for the abatement policy (and another corresponding set for the savings rate).

#### 4.1.3 Parametric Abatement Policy - Reduced Model

For analysis purposes and for comparison with a parametric stochastic abatement model, which we will introduce as part of our model extensions later, we define a parametric abatement model

$$\mu(t) = \min\left(\mu(0) + \frac{1 - \mu(0)}{T^{\mu=1}} t, 1.0\right), \tag{4}$$

where the parameter  $T^{\mu=1}$  represents the time for reaching 100% abatement. This model has only a single parameter, however it turns out that the optimal abatement policy in the full parameter model is comparably close to a functional form of the type (4).

#### 4.2 Cost

To characterize the resulting optimal mitigation paths, we utilise derived quantities. A prominent example of such a derived quantity is the *social cost of carbon*, SCC(t). The *SCC* is the time-*t marginal cost* of emitting one additional unit of carbon. Its unit is  $[SCC] = \frac{USD}{tCO_2}$ .

The "social cost of carbon" is maybe not a suitable measure to access the temporal distribution of the social burden, and hence intergenerational equity, because pricing emissions at the level of the SCC does not cover the cost associate with the climate mitigation paths: the accumulated value  $\int_0^T SCC(t) \cdot E(t) \cdot \frac{N(0)}{N(t)} dt$  does not match the accumulated cost  $\int_0^T C(t) \cdot \frac{N(0)}{N(t)} dt$ , see [44].

#### 4.2.1 Distribution of Cost

We analyse the temporal distribution of the cost C(t), which is the sum of the abatement cost  $C_A$  and the damage cost  $C_D$ 

$$C(t) := C_{\mathsf{A}}(t) + C_{\mathsf{D}}(t).$$

To gain a better understanding, we decompose how changes of the abatement policy  $\mu$  translate into changes of the cost structure and how these enter into the calibration's objective function.

#### 4.2.2 Sensitivity of Damage and Abatement Cost to Policy Changes

Differentiating the objective function (1) with respect to the abatement policy  $\mu$  gives

$$\frac{\mathrm{d}}{\mathrm{d}\mu} \int_0^T V(t) \frac{N(0)}{N(t)} \mathrm{d}t = \int_0^T \frac{\mathrm{d}V(t)}{\mathrm{d}C(t)} \frac{\mathrm{d}C(t)}{\mathrm{d}\mu} \frac{N(0)}{N(t)} \mathrm{d}t$$
$$= \int_0^T \frac{\mathrm{d}V(t)}{\mathrm{d}C(t)} \left(\frac{\mathrm{d}C_{\mathrm{D}}(t)}{\mathrm{d}\mu} + \frac{\mathrm{d}C_{\mathrm{A}}(t)}{\mathrm{d}\mu}\right) \frac{N(0)}{N(t)} \mathrm{d}t.$$
(5)

As the objective function depends on the cost, two transformations take place: First, the value *V* is defined as utility. Since the utility function is convex, there is some saturation, resulting in a over time decaying weight  $\frac{\partial V(t)}{\partial C(t)}$  applied to the cost. Second, the value is discounted. For the model of a constant positive discount rate, the discount factor constitutes an exponentially decaying weight  $\frac{N(0)}{N(t)}$  applied to the cost. The product of both weights represents the *cost-to-value-weight*.

The optimal abatement policy  $\mu$ , representing the equilibrium state of the model, fulfils

$$\frac{\mathrm{d}}{\mathrm{d}\mu} \int_0^T V(t) \frac{N(0)}{N(t)} \mathrm{d}t = \int_0^T \frac{\mathrm{d}V(t)}{\mathrm{d}C(t)} \left(\frac{\mathrm{d}C_{\mathsf{D}}(t)}{\mathrm{d}\mu} + \frac{\mathrm{d}C_{\mathsf{A}}(t)}{\mathrm{d}\mu}\right) \frac{N(0)}{N(t)} \mathrm{d}t \stackrel{!}{=} 0 \quad (6)$$

From eq. (6) we see why the model calibration may result in some intergenerational inequality: Let *s* be a fixed time at which a change in the abatement policy  $\mu(s)$  is considered. Since  $dC_D/d\mu(s) > 0$  and  $dC_A/d\mu(s) < 0$  we see from eq. (6) that the model equates the *increment* of the cost of damage to the *increment* of the cost of abatement, both weighted by the value-to-cost-sensitivity  $dV(t)/dC(t) \cdot N(0)/N(t)$ . We have

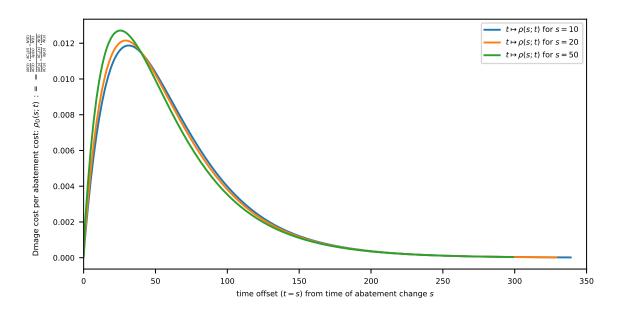
$$\int_0^T \frac{\mathrm{d}V(t)}{\mathrm{d}C(t)} \frac{\mathrm{d}C_{\mathrm{D}}(t)}{\mathrm{d}\mu(s)} \frac{N(0)}{N(t)} \mathrm{d}t + \int_0^T \frac{\mathrm{d}V(t)}{\mathrm{d}C(t)} \frac{\mathrm{d}C_{\mathrm{A}}(t)}{\mathrm{d}\mu(s)} \frac{N(0)}{N(t)} \mathrm{d}t \stackrel{!}{=} 0,$$
(7)

where the to cost function  $t \mapsto \frac{dC_A(t)}{d\mu(s)}$  and  $t \mapsto \frac{dC_D(t)}{d\mu(s)}$  have different temporal distributions.

A change in the abatement policy in time *s* affects the abatement cost  $C_A(s)$  only at time *s*, whereas it affects damage cost  $C_D(t)$  for possibly all  $t \ge s$ . This motivates to consider the damage cost per abatement cost induced by a change in the abatement policy  $\mu(s)$ , i.e.,  $\frac{\partial C_D(t)}{\partial \mu(s)} / \frac{\partial C_A(s)}{\partial \mu(s)}$ . As the model calibration is weighting cost by the cost-to-value weight, we consider the corresponding weighted damage-cost per abatement-cost sensitivity. The minus in (8) just accounts for the fact that abatement cost sensitivity and damage cost sensitivity have opposite signs.

$$\rho_{\mathsf{D}}(\boldsymbol{s};t) := -\frac{\frac{\partial V(t)}{\partial C(t)} \cdot \frac{\partial C_{\mathsf{D}}(t)}{\partial \mu(s)} \cdot \frac{N(0)}{N(t)}}{\frac{\partial V(s)}{\partial C(s)} \cdot \frac{\partial C_{\mathsf{A}}(s)}{\partial \mu(s)} \cdot \frac{N(0)}{N(s)}}.$$
(8)

From Equation (5) we see that for fixed *s* the integral over  $t \mapsto \rho(s; t)$  equals 1, i.e.,  $\rho_{D}(s)$  can be interpreted as a density. Hence, we may interpret  $\int_{0}^{\infty} t \cdot \rho_{D}(s; t) dt$  as the expected time of damage cost per avoided abatement cost in time *s*. In the



**Figure 8:** The density  $t \mapsto \rho_{\mathsf{D}}(s; t)$  for s = 10, 20, 50.

classical DICE 2016 model the expected time of damage cost per avoided abatement cost is approximately 70 years after the time of abatement (Figure 8), indicating that balancing marginal abatement and damage cost is a matter of intergenerational equity.

#### Sensitivity of Cost to Time of Reaching 100 % Abatement

The sensitivity  $\frac{d}{d\mu}$  is a functional derivative or gradient. To visualize the aggregated dependency structure, we consider the one-parameter deterministic abatement model (4). For this abatement model, the equilibrium state is given by

$$\int_{0}^{T} \frac{\mathrm{d}V(t)}{\mathrm{d}C(t)} \frac{\mathrm{d}C_{\mathrm{D}}(t)}{\mathrm{d}T^{\mu=1}} \frac{N(0)}{N(t)} \mathrm{d}t + \int_{0}^{T} \frac{\mathrm{d}V(t)}{\mathrm{d}C(t)} \frac{\mathrm{d}C_{\mathrm{A}}(t)}{\mathrm{d}T^{\mu=1}} \frac{N(0)}{N(t)} \mathrm{d}t \stackrel{!}{=} 0$$
(9)

Given the one-parameter model, we may depict the (weighted) damage cost sensitivity  $\frac{dC_D(t)}{dT^{\mu=1}}$  and the (weighted) abatement cost sensitivity  $\frac{dC_A(t)}{dT^{\mu=1}}$ , see Fig. 2.

#### 4.2.3 Cost per GDP

To measure the burden born by a generation, we consider the relative total damage and abatement cost per GDP. Putting nominal values relative to the GDP is a common measure, e.g., to assess national debt [45]. Here, it reflects that a more wealthy generation can carry a greater burden, mimicking an effort sharing scheme and also considering the greater capacity of future generations under the expected baseline growth in DICE.

In our numerical experiments we measure the instantaneous cost-per-GDP C(s)/GDP(s) as well as the lifetime-average cost-per-GDP  $\overline{C}(s)/\overline{GDP}(s)$ .

To allow for a netting inside a generation one may consider the (discounted) average value over a generation lifetime. For a given time *s* let  $T^{L}(s)$  denote the expected lifetime of a generation. We then consider the lifetime average cost and GDP.

$$\overline{C}(s) := \int_{s}^{s+T^{\mathsf{L}}(s)} C(t) \frac{N(s)}{N(t)} \mathrm{d}t, \qquad \overline{GDP}(s) := \int_{s}^{s+T^{\mathsf{L}}(s)} GDP(t) \frac{N(s)}{N(t)} \mathrm{d}t$$

#### 4.3 Model Extensions

We present modifications to integrated assessment models to investigate the effect of stochastic interest rates, funding of abatement costs, and non-linear financing cost [41]. These may be also incorporated into more complex IAMs. We use the simple DICE model to illustrate potential effects.

#### 4.3.1 Stochastic Interest Rates

The classical way in which interest rates are modelled in an IAM is via a discount factor. A time-*t* value V(t) is derived in the model, then discounted and aggregated to a final value forming the objective function

$$\int_0^T V(t) \exp\left(-\int_0^t r(s) \mathrm{d}s\right) \mathrm{d}t.$$

We model stochastic interest rates *r*. Our implementation allows to use a general discrete forward rate model (LIBOR Market Model). Our results were conducted with a classical Hull-White model. The model provides the stochastic numeráire *N* (and thus discount factor) as well as the stochastic forward rate *FR*. Setting the volatility of the interest rate to zero recovers the deterministic model with a possibly time-dependent interest rate (term-structure of interest rates). For exemplary parameterization, we set the mean reversion speed to a(t) = 0.02 and the volatility to  $\sigma(t) = 0.4\%$ . While our result should be interpreted as a qualitative result, showing that stochastic interest rates generate adverse effects, the interest rate scenarios generated by our parameter choice are realistic, rather conservative: the 1st and 99th percentile of the generated interest rates at t = 100 years are -1 % and 5 %, respectively. For a corresponding visualization, see [39].

If interest rates are stochastic but the function V(t) remains deterministic, adding stochastic interest rates does not change the interaction with the IAM since

$$\mathsf{E}\left(\int_{0}^{T} V(t) \exp\left(-\int_{0}^{t} r(s) \mathrm{d}s\right) \mathrm{d}t\right) = \int_{0}^{T} V(t) \mathsf{E}\left(\exp\left(-\int_{0}^{t} r(s) \mathrm{d}s\right)\right) \mathrm{d}t$$

$$= \int_{0}^{T} V(t) \exp\left(-\int_{0}^{t} \bar{r}(s) \mathrm{d}s\right) \mathrm{d}t,$$
(10)

with  $\bar{r}(t) = -\frac{\partial}{\partial t} \log \left( \mathsf{E} \left( \exp \left( - \int_0^t r(s) \mathrm{d}s \right) \right) \right).$ 

Due to the lack of a feedback that allows to adjust the abatement path and resulting damages, adding stochastic interest rates does not yet introduce changes in the model dynamics. This changes however, if one assumes that the abatement policy is adapted to the stochastic changes, here the change in interest rate level, i.e., if one allows  $\mu$  to be an (adapted) stochastic process.<sup>2</sup>

We use a classical Hull-White model, that is

$$dr(t) = (\theta(t) - a(t)r(t)) dt + \sigma(t) dW(t), \quad r(t_0) := r_0,$$
  

$$dN(t) = r(t) dt,$$
(11)

where an exact time-discretization scheme [47] was used. The model's meanreversion level  $\theta$  is such that the numéraire-relative zero coupon bond prices are martingales and their expectation agrees with the classical non-stochastic model. In other words, our stochastic interest rate model is such that the optimal *deterministic* abatement strategy would result in the same optimal emission pathway as the model with deterministic interest rates.

#### 4.3.2 Stochastic Abatement Policy

Assuming that a change in the interest rates triggers a re-calibration of the model, a stochastic interest rate implies a stochastic abatement policy, i.e.,  $\mu$  will become stochastic. The abatement policy  $t \mapsto \mu(t)$  becomes a stochastic process that adapts to the changes in interest rates. This reflects the possibility that the abatement policy can be adjusted to the interest rate scenarios.

The determination of an optimal stochastic abatement policy is a classical optimal exercise problem, as in the valuation of financial products with early exercise options in mathematical finance. Here,  $\mu$  takes the role of the exercise strategy. As the time-*t* optimal exercise is required to be a  $\mathcal{F}_t$ -measurable, it may be represented as a functional form of time-*t* measurable random variables, see [46].

An approach to determine the optimal exercise strategy is to parameterize  $\mu(t)$  as a functional form of  $\mathcal{F}_t$ -measurable random variables and apply a global optimization.

<sup>&</sup>lt;sup>2</sup>Our choice to make interest rates stochastic is only exemplary. The interest rate is an important factor in linking present abatement cost to the avoided future damage cost. While one may introduce stochasticity in many other state variables, the use of an abatement policy adapted to the economic factors (interest rates) will already lead to all other state variables becoming stochastic.

As we utilize a one-factor Markovian model for the short rate r, we consider a functional form where  $\mu$  depends on t and the current interest rate level r(t).

A simple example of a stochastic abatement model is a parametric one, where the abatement speed is a (linear) function of the interest rate level r, e.g.,

$$\mu(t,\omega) = \min(\mu_0 + (a_0 + a_1 \cdot r(t,\omega)) \cdot t, 1.0).$$
(12)

The simplicity of the models allows us to interpret the parameters. We have that  $a_0$  is the abatement speed at the zero interest rate level and  $a_1$  describes the change of the abatement speed by interest rate change,

$$a_0 = \frac{\partial \mu(t)}{\partial t}|_{r=0}, \quad a_1 = \frac{\partial}{\partial r} \frac{\partial \mu(t)}{\partial t}.$$

Obviously, the stochastic abatement models may achieve a larger-or-equal welfare or lower-or-equal welfare-risk than the corresponding deterministic model (4) simply due to the additional degree of freedom  $a_1$  in the abatement optimisation. The calibration of the optimal model (12) results in a negative value for  $a_1$ , reflecting that higher interest rate correspond to slower abatement speed.

#### 4.3.3 Funding of Abatement Costs

The geophysical part of the model models a global mean temperature level, which results from past emissions. The simple damage function of the DICE model provides damages as a reduction of global GDP. Damages may be reduced by performing abatement of emissions. Both abatement and damage are associated with costs. The time  $t_i$  costs are deduced from the time  $t_i$  GDP. The remainder is then available for consumption or investment. Investment adds to the capital which determines the GDP of the next time  $t_{i+1}$ . However, abatement cost and damage cost linked to the same abatement policy decision occur at significantly different times.

We introduce the option of funding of abatement cost. As abatement of emissions is a planned process of societal relevance, it is reasonable to assume that abatement cost are covered by a loan for which interest rate corresponds to the current discount rate. Our model allows to apply a funding spread, i.e. higher interest rates for loans than for discounting, but this is not considered in this general introduction of the phenomenon. In future work, the concept of the climate beta could be used to introduce an interest rate spread for abatement investments, as introduced in DICE-2023 [38].

We define  $C_{\mu}(t)$  as the instantaneous abatement costs in time *t*, i.e., the quantity that was formerly denoted by  $C_{A}(t)$ . We allow that  $C_{\mu}(t)$  might be funded for a period  $\Delta T_{A}$ . Thus abatement costs of  $C_{\mu}(t)$  are accrued with the forward rate  $FR(t, t + \Delta T_{A}; t)$  observed in *t*, such that the realized abatement cost  $C_{A}$  at time  $t + \Delta T_{A}$  are

$$C_{\mathsf{A}}(t + \Delta T_{\mathsf{A}}) := C_{\mu}(t) \left(1 + FR(t, t + \Delta T_{\mathsf{A}}; t)\Delta T_{\mathsf{A}}\right).$$
(13)

For the case of no funding, i.e.  $\Delta T_A = 0$ , we recover the classical model with  $C_{\mu} = C_A$ .

In a standard model for risk-neutral valuation, this change would have no effect, as the accruing is compensated by the discounting. However, the change might introduce an effect in the DICE model, due to the way how abatement and damage cost are associated and due to the time-preference included in the utility function.

Since damages occur instantaneously, we do not consider funding of these. The total cost is given by  $C(t) := C_A(t) + C_D(t)$ .

#### 4.3.4 Non-Linear Financing Costs

For an unsecured financial cash-flow its present value is defined by a discount factor times the cash-flow. If the future cash-flow is subject to default, the discount factor is lower, reflecting the additional value reduction due to the risk of (partial) default. As default is not an option for future damage cost, it appears as if the risk-free discount factor should apply. However, since no hedging strategy exists, a risk free funding is not possible. Thus additional cost may occur to secure the unsecured funding [41]. Since damage cost may become very large - much larger than funds provided by standard financial markets - it is reasonable to assume that these additional funding cost become (over-proportionally) large for larger cost. We model this by optionally adding non-linear financing cost using a non-linear funding model [41].

This model allows, that the discount factor may depend on the magnitude of the cash-flow. Thus the funding of larger cash-flows requires a premium to compensate for a larger default risk or other frictions. For our application, this means that larger cost get a larger weight.

Our model modification is now a modification of the damage cost. Let  $C_D^{\circ}(t)$  denote the damage cost of the classical model, i.e. the quantity that was formerly denoted by  $C_D(t)$ . We then re-define the effective damage costs as

$$C_{\mathsf{D}}(t) = C_{\mathsf{D}}^{\circ}(t) \cdot DC(C_{\mathsf{D}}^{\circ}(t); t),$$

where  $DC(C_D^{\circ}(t);t)$  is the *default compensation factor*. It is somewhat similar to the inverse of a discount factor, describing the over-proportional cost to fund large projects.

As  $C_{\rm D}^{\circ}(t)$  represents a time-value, it is natural that the default compensation factor depends on a normalized value only. We allow two different normalizations: either with the numéraire N(t) or with the GDP Y(t). Hence, our model for the default compensator is

$$DC(C_{D}^{\circ}(t);t) = DC^{N}(C_{D}^{\circ}(t);t) := DC^{*}(\frac{C_{D}^{\circ}(t)}{N(t)}),$$
 (14)

or, alternatively,

$$DC(C_{D}^{\circ}(t);t) = DC^{Y}(C_{D}^{\circ}(t);t) := DC^{*}(\frac{C_{D}^{\circ}(t)}{Y(t)}).$$
 (15)

The latter approach allows us to penalize damages that exceed a certain percentage of the GDP.

The factor  $DC^*(x)$  depends on the size of *x*. For small *x* we have  $DC^*(x) = 1$ , but for large *x* we may have factors > 1. Obviously, this will penalize large spikes in the costs.

In our numerical experiments the funding period was set to 20 years and the non-linear discounting was set to increase the default compensator significantly for damage cost over 3 % of the GDP (black). The parameter choice is exemplary, other experiments with similar results can be reproduced in the published source code.

As the abatement cost are usually smaller and part of a more planned process, we do not consider a default compensation factor for the abatement cost.

#### **Supplementary Material**

Supplementary material is available online.

## Code and Data availability

The model code for the stochastic DICE model, including the proposed extensions, as well as the experiments generating the data and figures, is available in the following Git repository https://gitlab.com/finmath/finmath-climate-nonlinear.

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### **Author Contribution**

C.F. and L.Q. developed the research idea together. C.F. lead the re-implementation of the DICE model and extensions, as well as the experiments with input from L.Q. L.Q. lead the plotting implementation with input from C.F. C.F. and L.Q. analyzed and interpreted results. C.F. derived the analytic expression for generational inequality. C.F and L.Q. wrote the manuscript.

### **Competing Interests**

The authors declare that they have no competing interests.

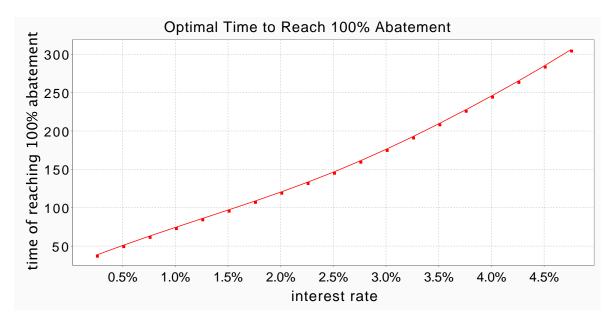
Supplementary information for

# Intergenerational Equitable Climate Change Mitigation: Negative Effects of Stochastic Interest Rates; Positive Effects of Financing

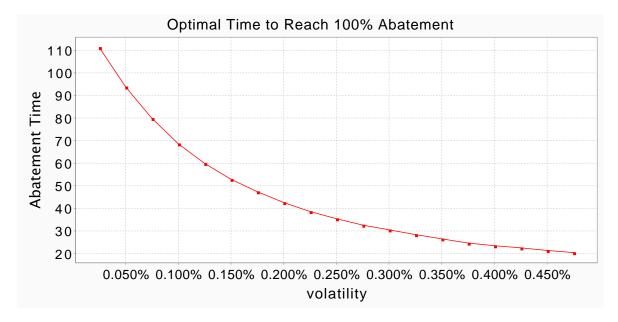
Christian P. Fries o and Lennart Quante o

# Supplementary figures

Supplementary figure 1	Sensitivity of Abatement to Interest Rate Level
Supplementary figure 2	Sensitivity of Abatement to Interest Rate Volatility



**Supplementary Figure 1** Increasing interest rate level increases the time of 100% abatement  $T^{\mu=1}$ .



**Supplementary Figure 2** Increasing interest rate volatility, i.e. riskiness, decreases the time of 100% abatement  $T^{\mu=1}$ .